Mesh Analysis

Objective:
To analyze circuits using a systematic technique: the mesh analysis.

Mesh Analysis
The mesh analysis is a systematic way of applying KVL around each mesh of a circuit and describes the branch voltages in terms of the mesh currents. This will give us a set of equations that we solve together to find the mesh currents. Once we find the mesh currents we can use them to calculate any other currents or voltages of interest.

Case #1: Circuits with Independent voltage Sources.

Example #1:
For following circuit find the mesh currents and use the mesh currents to find $i_x$.

Solution
Step #1
Identify all of the meshes.

Step #2
Assign currents to all of the meshes
Step #3
Apply the KVL around each mesh. In this step for each mesh we describe the branch voltages in term of mesh currents.

Apply KVL around mesh # 1

\[-7 + 1 \, i_1 + 2 \, (i_1 - i_2) + 6 = 0\]

\[3 \, i_1 + 2 \, i_2 = 1 \quad (1)\]

Apply KVL around mesh # 2

\[3 \, i_2 + 4 \, i_2 + 9 - 6 + 2 \, (i_2 - i_1) = 0\]

\[-2 \, i_1 + 9 \, i_2 = -3 \quad (2)\]

Step #4
Solving question 1 and 2 for the unknown mesh currents \((i_1\) and \(i_2\)):

\[i_1 = 0.13 \, A\]

\[i_2 = -0.304 \, A\]

\[i_x = i_1 - i_2 = 0.434 \, A\]
Example #2:
Write the mesh (loop) equations for the following circuit and then find $i_x$, $i_y$ and $v$

![Circuit Diagram]

Solution:

Apply KVL around mesh # 1

\[ 6 + 4 i_1 + 6 (i_1 - i_3) = 0 \]
\[ 10 i_1 - 6i_3 = -6 \quad \ldots \quad (1) \]

Apply KVL around mesh # 2

\[ 9 i_2 - 6 + 3 (i_2 - i_3) = 0 \]
\[ 12 i_2 - 3i_3 = 6 \quad \ldots \quad (2) \]
Apply KVL at mesh # 3

\[ 3(i_3 - i_2) + 6(i_3 - i_1) + 12i_3 = 0 \]
\[ -6i_1 - 3i_2 + 21i_3 = 0 \ldots (3) \]

Solve the three equations

\[ i_1 = -0.6757 \, A, \; i_2 = 0.4685 \, A, \; i_3 = -0.1261 \, A \]
\[ i_x = i_1 - i_2 = -1.1442 \, A \]
\[ i_y = i_3 - i_2 = -0.5946 \, A \]
\[ v = -9i_2 = 4.2165 \, V \]

Case #2: Circuits with Independent Current Sources

Example #3:

Find \( V_0 \) by using the mesh analysis

Solution
It is clear that:

\[ i_1 = 4 \text{ mA} \]
\[ i_2 = -2 \text{ mA} \]

Apply KVL around mesh # 3

\[-3 + 4k \ (i_3 - i_2) + 2k \ (i_3 - i_1) + 6k \ i_3 = 0\]

\[ i_3 = \frac{1}{4} \text{ mA} \]

To find \( V_0 \):

\[ +3 - 6 \ (i_3) + V_0 = 0 \]

\[ V_0 = -\frac{3}{2} \]
Case #3: Circuits with a current source common to two meshes

Example #4:

Solution
Apply KVL around the mesh #1

\[ 2i_1 + 3(i_1 - i_3) + 1(i_1 - i_2) = 0 \]
\[ 6i_1 - i_2 - 3i_2 = 0 \quad \text{(1)} \]

**Supermesh:**

When a current source is common to two meshes we use the concept of supermesh. A supermesh is created from two meshes that have a current source in common as shown in the above figure.

Apply KVL around the supermesh

\[ 7 - 1(i_2 - i_1) + 3(i_3 - i_1) + 1i_3 = 0 \]
\[ -4i_1 + i_2 + 4i_3 = 7 \quad \text{(2)} \]

We have two equations with three unknown variables!!!. The third equation can be obtained by using the relation between the \( i_2 \) and \( i_3 \) as follows:

\[ i_3 - i_2 = 7A \quad \text{(3)} \]

Solve the three equations

\[ i_1 = 1.5 \, A, \quad i_2 = -3 \, A, \quad i_3 = 4 \, A \]

**Case #4: Circuits with dependent sources**

For circuits that include dependent sources, first we ignore the fact that the dependent source is a dependent source and we write the mesh-current equations as we would for a circuit with independent sources. The mesh-current equations will have extra unknown variables for the dependent sources beside to the normal unknown mesh currents. All the extra unknown variable of the dependent sources must be described in term of the mesh currents. There MUST be a relation between the unknown variable of the dependent and the mesh currents, because mesh currents can be used to describe any current or voltage of any branch in the circuit. The following examples will show you how to apply the mesh analysis for circuits with dependent sources.
Example #5:
Find $V_o$ by using mesh analysis.

Solution

From mesh #1 \[ i_1 = \frac{V_x}{2} \ldots \ldots (1) \]

From mesh #2 \[ i_2 = 2A \ldots \ldots (2) \]

Apply KVL around mesh #3

\[-5 + (i_3 - i_1) + 2 + 6i_3 = 0 \ldots \ldots (3)\]
Equation (1) has an extra unknown variable \(v_x\). We should relate the extra unknown variable of the dependent source to the mesh currents. From the above figure it is clear that:

\[ V_x = 4(i_1 - i_2) \]

In Eq.(1)

\[ i_1 = \frac{4(i_1 - i_2)}{2} \]

\[ i_1 - 2i_2 = 0 \ldots (4) \]

From Eq.(2) and Eq.(4)

\[ i_1 = 4 \, A \]

From Eq.(3)

\[ i_3 = \frac{11}{8} \, A \]

\[ V_o = i_3 \times 6 = \frac{33}{4} \]

**Example #6:**

For the following circuit find the mesh currents.

\[
\begin{align*}
4 & \quad 6 \\
20 & \quad \text{+} & \quad \text{-} & \quad V_x \\
\text{+} & \quad \text{V}_x & \quad \text{-} & \quad 2 \\
4 & \quad V_x & \quad 4
\end{align*}
\]

**Solution**
Apply KVL around the supermesh:

\[-20 + 4i_1 + 6i_2 + 2i_2 = 0\]
\[4i_1 + 8i_2 = 20 \quad \ldots \quad (1)\]

We have one equation with two unknown variables!!! The second equation can be obtained by using the relation between the \(i_1\) and \(i_2\) as follows:

\[i_2 - i_1 = \frac{v_x}{4} \quad \ldots \quad (2)\]

Equation (2) has an extra unknown variable \(v_x\). We should relate the extra unknown variable \(v_x\) of the dependent source to the mesh currents. From the above figure it is clear

\[v_x = 2i_2\]

In Eq.(2)

\[i_2 - i_1 = \frac{2i_2}{4}\]
\[2i_1 - i_2 = 0 \quad \ldots \quad (3)\]

Solving Eq.(1) and Eq.(3)

\[i_1 = 1, i_2 = 2\]